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**GARISSA UNIVERSITY**

**UNIVERSITY EXAMINATION 2018/2019 ACADEMIC YEAR FOUR**

**SECOND SEMESTER EXAMINATION**

**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**

**FOR THE DEGREE OF BACHELOR OF EDUCATION**

**COURSE CODE: MAT 410**

**COURSE TITLE: RINGS AND MODULES**

**EXAMINATION DURATION: 2 HOURS**

**DATE: 12/02/2020 TIME: 09.00-11.00 AM**

**INSTRUCTION TO CANDIDATES**

* **The examination has FIVE (5) questions**
* **Question ONE (1) is COMPULSORY**
* **Choose any other TWO (2) questions from the remaining FOUR (4) questions**
* **Use sketch diagrams to illustrate your answer whenever necessary**
* **Do not carry mobile phones or any other written materials in examination room**
* **Do not write on this paper**

**This paper consists of THREE (3) printed pages *please turn over***

**QUESTION ONE (COMPULSORY)**

1. If $ R $ is a system satisfying all the conditions of a ring with unit element with possible exception of $x+y=y+x,$ prove that the axiom $x+y=y+x$ must hold in $R$ and that $R$ is a ring. **(5 Marks).**
2. Define a nilpotent element of a ring $R$ and use a ring of $M\_{2}(R)$ of matrices with real entries to illustrate this. (2 Marks).
3. Prove that a commutative ring $ R$ with unity is an integral domain exactly when for $a\ne 0$, $ a\in R$, $a.b=a.c $implies that $b=c $, $b,c\in R$ (4 Marks).
4. Give the definition of a left and right ideal and use it show that for a subring $A=\left\{\left[\begin{matrix}a&b\\0&0\end{matrix}\right] :a,b \in R \right\}$ of the matrix ring $M\_{2}(R)$ , it is a right ideal but not a left ideal. (6 Marks).
5. Let $ R=\left\{\left[\begin{matrix}x&y\\y&x\end{matrix}\right] :x, y\in Z \right\}$ and $ φ$ be the mapping that takes $\left[\begin{matrix}x&y\\y&x\end{matrix}\right]$ to $ x-y$ , show that the mapping $φ$ is a homomorphism and hence find its kernel. (7 Marks).
6. Prove that if $R$ is a ring with unit element$,e$, then $R$ is of characteristic $n$ if and only if $ne=0$ and that $n$ is the smallest positive integer. (6 Marks).

**QUESTION TWO (20 MARKS)**

1. Show that $S=\{a+b\sqrt{2} :a, b\in Z \}$ for the operations$ +$, $×$ is an integral domain but not a field. (12 Marks).
2. If R is a ring such that $a^{2}=a$ for all $ a\in R$ prove that
3. $a+a=0 $all $ a\in R$ (3 Marks)
4. $a+b=0 ⇒a=b $. (2 Marks)
5. $R$ is a commutative ring. (3 Marks)

**QUESTION THREE (20 MARKS)**

1. Define a ring homomorphism and use it to show that if $R\_{1}=\left\{\left[\begin{matrix}a&0\\0&0\end{matrix}\right] :a \in R \right\}$ where $R$ is a ring and the mapping $ϕ:R\_{1}⟶ R$ defined by $ ϕ\left(\left[\begin{matrix}a&0\\0&0\end{matrix}\right]\right)=a$ for all $\left[\begin{matrix}a&0\\0&0\end{matrix}\right]\in R\_{1}$ then $ϕ $ is an isomorphism. (6 Marks).
2. Prove that if $ϕ:R\_{1}⟶ R^{'}$ is a homomorphism with kernel S, then S is an ideal of $R $ (6 Marks).
3. Show that every finite integral domain is a field. (8 Marks).

**QUESTION FOUR (20 MARKS)**

1. Prove that for two ideals $ A$ and $B$ of a ring $R$ , $A\bigcup\_{}^{}B$ is an ideal of $R$ if and only if either $A⊆B$ or $B⊆A$ . (6 Marks).
2. Find the principal ideals of the ring $\left〈Z\_{6} , +\_{6},×\_{6}\right〉$ (6 Marks).
3. In the ring $Z\left[i\right], $show that $ I=\{a+bi \in Z[i] :a, b are both even\}$ is an ideal of $Z[i]$ (8 Marks).

**QUESTION FIVE (20 MARKS)**

1. By considering the ring $^{Z}/\_{nZ}$ where $n>1$ prove that $a$ is a unit if and only if $gcd(a,n)=1$ (5 Marks).
2. Find the unit elements and zero divisors in the ring given in (a) above if $n=12$ (7Marks).
3. Use division algorithm to find the $ gdc(204,135)$.Hence or otherwise, express this $ gcd(a, n)=d$ in the form $d=204u + 135v$. (8 Marks).