# GARISSA UNIVERSITY COLLEGE 

(A Constituent College of Moi University)

# UNIVERSITY EXAMINATION $2016 / 2017$ ACADEMIC YEAR ONE SECOND SEMESTER EXAMINATION <br> SUPPLEMENTARY/SPECIAL EXAMINATION <br> SCHOOL OF BUSINESS AND ECONOMICS <br> FOR THE DEGREE OF BACHELOR OF BUSINESS MANAGEMENT 

COURSE CODE: BBM 115
COURSE TITLE: BUSINESS MATHEMATICS II

## EXAMINATION DURATION: 3 HOURS

DATE: 25/09/17
TIME: 09.00-12.00 PM

## INSTRUCTION TO CANDIDATES

- The examination has SIX (6) questions
- Question ONE (1) is COMPULSORY
- Choose any other THREE (3) questions from the remaining FIVE (5) questions
- Use sketch diagrams to illustrate your answer whenever necessary
- Do not carry mobile phones or any other written materials in examination room
- Do not write on this paper


## QUESTION ONE (COMPULSORY)

(a) Define the following terms as used in business:

Consumers' surplus (1 mark)
Producers' surplus(1 mark)
Loan amortization(1 mark)
(b) The output, Q for a firm over time, t in years is given by the function

$$
Q=\frac{t^{3}}{30}-\frac{t^{2}}{5}+\frac{3 t}{10}+120
$$

Determine the years in which the output is at maximum and minimum. ( $\mathbf{5}$ marks)
(c) The production function, Q of a commodity is given by

$$
Q=10 L+5 K-L^{2}-2 K^{2}+3 K L
$$

Where: L is the labour input
i. K is the capital input

Find the marginal product of labour (MPL) and marginal product of capital (MPK) when $\mathrm{L}=1$ and $\mathrm{K}=2$.
(d) The marginal cost (MC) function is given by $M C=18+12 Q-9 Q^{2}$ and $T C=100$ when $Q=0$. Find the total cost (TC ) function. ( $\mathbf{3}$ marks)
(e) The demand and supply function curves for a good are given respectively by the equations:

$$
P=300 e^{-0.2 \varphi} \text { and } P=2 e^{0.8 Q}
$$

Find the equilibrium price P and quantity Q .
(f) A manufacturer makes two products $x_{1}$ and $x_{2}$. The first requires 5 hours for processing, 3 hours for assembling and 4 hours for packaging. The second requires 2 hours for processing, 12 hours for assembling and 8 hours for packaging. The plant has 40 hours available for processing, 60 hours for assembling and 48 hours for packaging. The profit margin for $x_{1}$ is $\$ 7$ and for $x_{2}$ it is $\$ 21$.Express the data in equations and inequalities necessary to determine the output mix that will maximize profits.
(6 marks)

## QUESTION TWO (15 MARKS)

The demand function for a good is given by $P=50-2 Q$, while the total $\operatorname{cost}(\mathrm{TC})$ is given by $T C=160+2 Q$, where P is the price and Q is the quantity

## Required:

i. Write down expressions for the total revenue (TR) and the profit function ( $\pi$ )
ii. Sketch the TC and TR functions on the same diagram( 4 marks)
iii. Find algebraically, in terms of Q , when the firm breaks even, makes a profit and makes a loss. ( 5 marks)
iv. Determine the maximum profit and the value of Q at which profit is maximum. ( 4 marks)

## QUESTION THREE

(a) A firm employing labour $(\mathrm{L})$ as the only factor input has the following production function.
$Q=f(L)=L e^{-0.2 L}$

## Required:

i. Find the critical value of L.(3 marks)
ii. Confirm that the critical value of $L$ maximizes $Q$.
(4 marks)
(b) The total revenue and total cost of a multinational firm are given as follows:

$$
\begin{aligned}
& T R=12 \operatorname{In}(Q+1) \\
& T C=4 Q
\end{aligned}
$$

## Required:

i. Find the profit ( $\pi$ ) function.
(1 marks)
ii. Determine the critical values of Q for the profit function.( $\mathbf{3}$ marks)
iii. Confirm that the critical value maximizes profit (4 marks)

## QUESTION FOUR(15 MARKS)

(a) Find the cross-partial derivatives of the function $z=x^{3} y^{4}$ ( $\mathbf{3}$ marks)
(b) For a multinational company, the number of units produced when using $x$ units of labour and $y$ units of capital is given by

$$
f(x, y)=80 x^{\frac{1}{4}} y^{\frac{3}{4}}
$$

## Required:

i. Find the equations for both marginal productivities (MPL and MPK)
(2 marks)
ii. Evaluate and interpret the results in (i) when 625 units of labour and 81 units of capital are used. (4 marks)
(c) A production function is given by the equation $Q=8 K^{\frac{1}{2}} L^{\frac{1}{4}}$ where K is the capital input and L the labour input.

## Required:

i. Find the marginal production of labor and marginal production of capital.(2 marks)
ii. Determine whether or not the function is characterized with diminishing returns to factor inputs. (4 marks)

## QUESTION FIVE (15 MARKS)

(a) Evaluate $\int_{1}^{3} \frac{1}{2} x^{2} d x$
(2 marks)
(b) The marginal cost function is given by the equation $M C=15+10 Q-6 Q^{2}$. Find the total cost function (TC) given that $\mathrm{TC}=50$ when $\mathrm{Q}=2$.
(c) The supply function is given by the equation $P=Q^{2}+6 Q$ where Q is the quantity of goods.

## Required:

Calculate the producers' surplus whenthe market equilibrium price $P_{e}=40$ units. Graph the the supply function and shade the producers' surplus.
(d) The demand function is given by the equation $P=\frac{100}{Q+2}$ where Q is the quantity of goods.

## Required:

i. Calculate the consumers' surplus when the market equilibrium price $P_{e}=20$ units. Graph the the demand function and shade the consumers' surplus.

## QUESTION SIX (15 MARKS)

(a) To provide for future education costs, a family considers various methods of savings. The savings will continue for a period of 10 years at an interest rate of $7.5 \%$ p.a.

## Required:

(i)Calculate the value of the fund at the end of 10 years when a single deposit of $\$ 2000$ is made annually.
(ii) How much should be deposited each year if the final value of the fund is $\$ 40000$. ( $\mathbf{2}$ marks)
(iii) How much should be deposited each month if the final value of the fund is $\$ 40000$. (3 marks)
(b) Halima intends to amortize a loan of $\$ 10000$ at a rate of $7 \%$ p.a in six years. She decides to make annual equal payments at the end of each year.

## Required:

(i)How muchwill Halima pay annually?
(ii) Prepare a loan amortization schedule for the loan?(6 marks)

